14-5 Planes

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SECTION C

Series and differential equations

1. *[Maximum mark: 9]*

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y+2}{xy+1}$ and $y = 1$ when $x = 0$, use Euler's method with interval $h = 0.5$ to find an approximate value of y when $x = 1$. *[9 marks]*

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A plane can be specified in any of the following ways:

3 non-collinear Points

2 Intersecting lines

A point and a vector that the plane is perpendicular to

A specified distance from the origin & a vector that that the plane is perpendicular to

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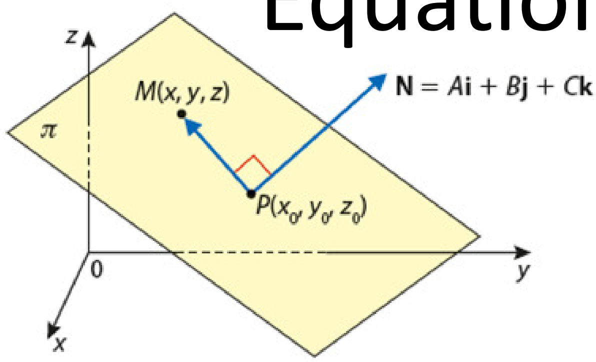
Equations of Planes



Let N be a vector perpendicular to the plane called the normal vector. Let $P(x_0,y_0,z_0)$ be a point on the plane. Let M (x, y, z) be any point on the plane.

$$\overrightarrow{PM} \cdot \overrightarrow{N} = 0$$

$$\overrightarrow{PM} = < x - x_0, y - y_0, z - z_0 >$$

$$\overrightarrow{N} = < A, B, C >$$

$$\overrightarrow{PM} \cdot \overrightarrow{N} = < x - x_0, y - y_0, z - z_0 > \cdot < A, B, C >$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

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Cartesian Equation of a Plane

The Cartesian equation of a plane through the point $P(x_0,y_0,z_0)$ which has a normal vector $\overrightarrow{N} = Ai + Bj + Ck$ is

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

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Ex1. Given a plane contains the point A(-1,1,2) and is normal to the vector n = 2i – j – k.

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

a.) Find the equation of the plane.

$$2(x+1)-1(y-1)-1(z-2)=0$$

$$2x+2-y+1-z+2=0$$

$$x-y-z=-5$$

b.) Hence, show that the point B(0,4,1) lies on the plane.

$$0-4-1=-5$$

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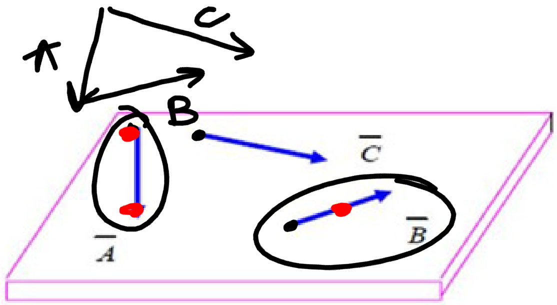

Three coplanar vectors $\vec{a}, \vec{b}, \vec{c}$ are given. If $\vec{a}$ is not parallel to $\vec{b}$, then $\vec{c}$ can always be expressed as a linear combination of $\vec{a}$ and $\vec{b}$.

It is always possible to find two scalars α and β such that $\vec{c} = \alpha \cdot \vec{a} + \beta \cdot \vec{b}$

1 1.3

To define the orientation of a plane uniquely, we require either:

2 non-parallel vector which both lie in the plane.

One vector perpendicular (normal) to the plane. ✓

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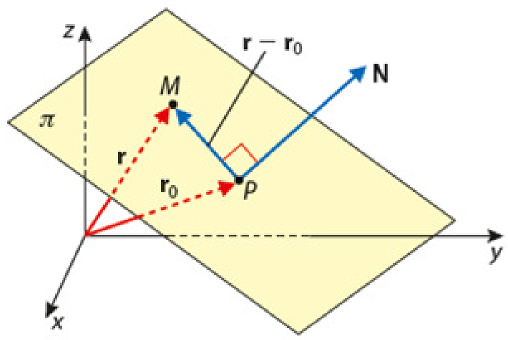
Equations of Planes



Suppose the point r(x,y,z) lies in the plane containing a known point r_0 (x_0, y_0, z_0) and 2 non-parallel vectors u and v.

$r_0 r$

$$\overline{rr_0} = \alpha \cdot \bar{u} + \beta \cdot \bar{v}$$

$r - r_0$

$$r_0 r = \alpha \cdot \bar{u} + \beta \cdot \bar{v}$$

$$r = r_0 + \alpha \cdot \bar{u} + \beta \cdot \bar{v}$$

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Vector Equation of a Plane

r = Any point on the plane

r_0 = A known point on the plane

$\vec{u}$ and $\vec{v}$ are two non-parallel vectors that lie in the plane

α and β are two independent real parameters

$$r = r_0 + \alpha \cdot \vec{u} + \beta \cdot \vec{v}$$

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Ex2. Given the points A(1,0,1), B(-1,1,0), and C(0,1,-1). Find the equation of the plane ABC

a.) in vector form

b.) in parametric form

c.) in Cartesian form

vector

a.) $(x,y,z) = (1,0,1) + \alpha\langle -2,1,-1\rangle + \beta\langle -1,1,-2\rangle$

b.) parametric

$x = 1 + -2\alpha + -1\beta$

$y = 0 + 1\alpha + -2\beta$

$z = 1 + -1\alpha + -2\beta$

c.)

$$n = \begin{vmatrix} i & j & k \\ -2 & 1 & -1 \\ -1 & 1 & -2 \end{vmatrix} = i\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} - j\begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix} + k\begin{vmatrix} -2 & 1 \\ -1 & 1 \end{vmatrix}$$

$\langle x-1, y, z-1 \rangle$

$n = i(-2+1) - j(4-1) + k(-2+1)$

$n = -1i - 3j - 1k$

$\langle -1, -3, -1 \rangle$

$-1(x-1) + 3y - 1(z-1) = 0$

$-x + 1 - 3y - z + 1 = 0$

$-x - 3y - z + 2 = 0$

$x + 3y + z - 2 = 0$

$\boxed{x + 3y + z = 2}$ Cartesean

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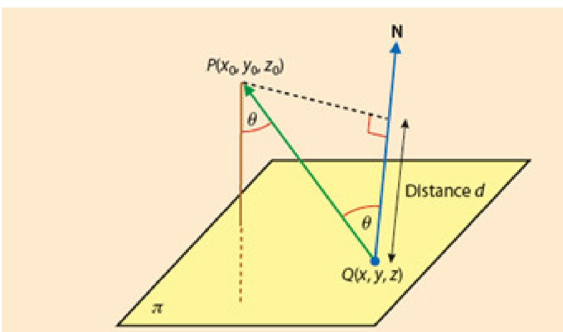
Distance Between a Point and a Plane



The distance between a point $P(x_0,y_0,z_0)$ and a plane π with equation

$Ax + By + Cz = D$ is given by:

$$d = \frac{|Ax_0 + By_0 + Cz_0 - D|}{\sqrt{A^2 + B^2 + C^2}}$$

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Ex3. Find the shortest distance from the point A (2,-1,3) to the plane x – y + 2z = 27.

$$\frac{2+1+6-27}{\sqrt{1^2+(-1)^2+2^2}} = \frac{18}{\sqrt{6}}$$

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Ex4. Find the distance between the parallel planes

$3x - 2y + 4z = 9$ $-6x + 4y - 8z = 12$

$(1, -1, 1)$

$$d = \frac{|(1\cdot -6) + (-1\cdot 4) + (1\cdot -8) - 12|}{\sqrt{36+16+64}}$$

$$\frac{30}{\sqrt{116}}$$

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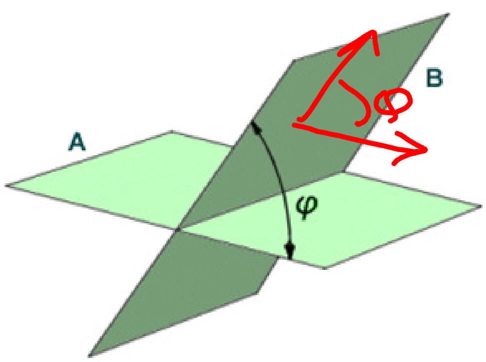
The Angle Between 2 Planes



Consider 2 Planes A and B with normals $\overrightarrow{N_1}$ and $\overrightarrow{N_2}$ respectively. The acute angle between planes A and B is given by:

$$\cos\varphi = \frac{\left|\overrightarrow{N_1}\cdot\overrightarrow{N_2}\right|}{\left|\overrightarrow{N_1}\right|\left|\overrightarrow{N_2}\right|}$$

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Ex5. Find the acute angle between the planes with equations x + y – z = 8 and 2x – y + 3z = -1.

$\vec{N_1} = <1, 1, -1> \quad \vec{N_2} = <2, -1, 3>$

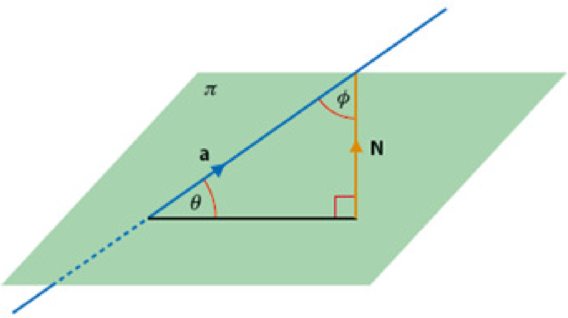
$\vec{N_1} \cdot \vec{N_2} = 2 - 1 - 3 = -2$

$|\vec{N_1}| = \sqrt{1+1+1} = \sqrt{3} \quad |\vec{N_2}| = \sqrt{4+1+9} = \sqrt{14}$

$\cos\varphi = \frac{|-2|}{\sqrt{3} \cdot \sqrt{14}} = \frac{2}{\sqrt{42}}$

$\varphi = 72.0°$

The Angle between a Line and a Plane



The acute angle θ between a line ℓ with direction vector $\vec{a}$ and plane π with normal vector $\vec{N}$ is given by

$$\sin\theta = \frac{\left|\vec{a}\cdot\vec{N}\right|}{\left|\vec{a}\right|\left|\vec{N}\right|}$$

Ex6. Find the angle between the plane x + 2y – z = 8 and the line with equations:

$\langle 1, 2, -1 \rangle$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

x = t

y = 1 – t

z = 3 + 2t

$\sin^{-1} \frac{|1-2-2|}{\sqrt{6}\sqrt{6}} = \sin^{-1}\left(\frac{3}{6}\right) = 30°$

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1,3,6,8,10,15,16,18,20,21,23-31
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